CALCULUS 233H FINAL EXAM

This exam is worth 100 points, with each problem worth 20 points. Please complete Problem 1 and then *any four* of the remaining problems. Unless indicated, you must show your work to receive credit for a solution. Make sure you answer every part of every problem you do.

When submitting your exam, please indicate which problems you want graded by writing them in the upper right corner on the cover of your exam booklet. You must select exactly four problems; any unselected problems will not be graded, and if you select more than four only the first four (in numerical order) will be graded.

(1) Please classify the following statements as *True* or *False*. Write out the word completely; do not simply write T or F. There is no partial credit for this problem, and it is not necessary to show your work for this problem.
  (a) If $f(x, y, z) = x^2 + y^2 + z^2$, then $\text{curl}(\nabla f) = \mathbf{0}$.
  (b) If $\mathbf{F} = \nabla f$, then $\oint_C \mathbf{F} \cdot d\mathbf{r} = 0$ for any simple closed curve C.
  (c) If $f(x, y)$ satisfies $f_{xx} f_{yy} - (f_{xy})^2 < 0$ at the point $(1, 2)$, then $(1, 2)$ is a saddle point for the graph of f.
  (d) In polar coordinates, $dA = r\, dr\, d\theta$.
  (e) If $f(x, y) = e^x \sin(y)$, then $f_{xy} = f_{yx}$.

(2) (a) Find an equation of the plane P through the origin and perpendicular to the vector $(1, 1, 1)$.
  (b) Find parametric equations for the line ℓ through the points $(1, 1, 1)$ and $(0, 0, 3)$.
  (c) Find the distance between P and ℓ.

(3) Let C be the circle $\{x^2 + y^2 = R^2\}$, where R is a constant.
  (a) Show that the curvature of C at any point is $1/R$.
  (b) What is the torsion of C? Why?

(4) Find and classify the critical points of $f(x, y) = 4 + x^3 + y^3 - 3xy$.

(5) Compute $\int_C (2y^2 - 12x^3y^3)dx + (4xy - 9x^4y^2)dy$ where C is
  (a) the line segment from $(0, 0)$ to $(1, 1)$
  (b) the parabolic path along $y = x^2$ from $(0, 0)$ to $(1, 1)$.

(6) Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = y\mathbf{i} + xz\mathbf{j} + z^2\mathbf{k}$, and where C is given by
  (a) $\mathbf{r}(t) = (t, t, t)$, and $0 \le t \le 1$.
  (b) $\mathbf{r}(t) = (t, t^2, t^3)$, and $0 \le t \le 1$.

(7) Compute $\oint_C (y^2 - \ln x)dx + (3x + \sin y)dy$, where C is the boundary of the region bounded by $y = x^2$ and $y = 4$, and C is traversed *counterclockwise*.

(8) Find the volume of the solid under the plane $z = 1$ and above the paraboloid $z = x^2 + y^2$.

(9) Compute $\iiint_R x\, dV$, where R is the region bounded by the graphs of $x \ge 0$, $y \ge 0$, $z \ge 0$, $x + y = 1$, and $z = 1 - y^2$.

HAVE A GOOD HOLIDAY

Date: Wednesday, 17 December 2003.