CALCULUS 233H FINAL EXAM

This exam is worth 100 points, with each problem worth 20 points. Please complete Problem 1 and then *any four* of the remaining six problems. Unless indicated, you must show your work to receive credit for a solution.

When submitting your exam, please indicate which problems you want graded by writing them in the upper right corner on the cover of your exam booklet. You must select exactly four problems; any unselected problems will not be graded, and if you select more than four only the first four (in numerical order) will be graded.

(1) Please classify the following statements as *True* or *False*. Write out the word completely; do not simply write T or F. There is no partial credit for this problem, and it is not necessary to show your work for this problem.
  (a) In polar coordinates, the infinitesmal element of area for double integrals is $dA = r\,dr\,d\theta$.
  (b) If $\mathbf{F}$ is a vector field with $\operatorname{curl}\mathbf{F} = \mathbf{0}$, then $\mathbf{F}$ is conservative.
  (c) An equation for the tangent plane to the graph of $z = y^2 - x^2$ at $(-4, 5, 9)$ is given by $8x + 10y - z = 9$.
  (d) The distance between the parallel planes $P_0 = \{x+y+z = 0\}$ and $P_1 = \{x+y+z = 1\}$ is $\sqrt{3}/3$.
  (e) If $f(x, y)$ satisfies $f_{xx}f_{yy} - (f_{xy})^2 < 0$ at a point, then that point is a saddle point on the graph of f.

(2) Compute the curvature and torsion at $t = 0$ of the space curve $\mathbf{r}(t) = (t\sqrt{2}, e^t, e^{-t})$.

(3) Find the global maximum and global minimum of $f(x, y) = x^2 + xy + y^2$ on the closed triangle with vertices $(1, 1)$, $(1, -1)$, $(0, 0)$.

(4) Find the volume of the solid region between the graphs of $f(x, y) = 2 - x^2 - y^2$ and $g(x, y) = x^2 + y^2$.

(5) Compute $\int \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = x\mathbf{i} + y^2\mathbf{j} + z^3\mathbf{k}$, and $\mathbf{r}(t) = (\cos(t), \sin(t), t)$ from $t = 0$ to $t = 2\pi$.

(6) Compute $\oint_C (e^x + 2y)\,dx + (3x - \tan y)\,dy$, where C is the circle of radius one at the origin, and the integral is taken in the clockwise direction.

(7) Suppose that f and g are functions of x, y, z. Compute $\operatorname{div}(\nabla f \times \nabla g)$. (Your answer will be expressed in terms of derivatives of f and g, e.g. f_x, f_{xy}, etc.)

Date: Tuesday, 17 December 2002.