CALCULUS 233H EXAM II

This exam is worth 100 points, with each problem worth 20 points. Please complete Problem 1 and then *any four* of the remaining six problems. Unless indicated, you must show your work to receive credit for a solution.

When submitting your exam, please indicate which problems you want graded by writing them in the upper right corner on the cover of your exam booklet. You must select exactly four problems; any unselected problems will not be graded, and if you select more than four only the first four (in numerical order) will be graded.

(1) Please classify the following statements as *True* or *False*. Write out the word completely; do not simply write T or F. There is no partial credit for this problem, and it is not necessary to show your work for this problem. In this problem f is a function of two variables x and y.

  (a) If $f = x^2y + xy^3$, then $f_{xy} = f_{yx}$.

  (b) If f is continuous on a region R in the xy-plane, then f attains a global maximum and a global minimum.

  (c) If R is a region in the xy-plane, and the domain of f contains R, then $\int\int_R f\,dA$ is the volume under the graph of f and above R.

  (d) The directional derivative of $f = x^2 + y^2$ in the direction of $(1, 0)$ is $2x$.

  (e) If z is a function of r and s, and r and s are each functions of u and v, then $z_r = z_u r_u + z_v v_r$.

  (f) If $f_{xx}f_{yy} - (f_{xy})^2 = 0$ at a point, then that point is a saddle point on the graph of f.

(2) Compute the volume of the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 3)$.

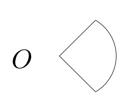
(3) Find the global maximum and global minimum of $f(x, y) = 2x^2 - y^2 - xy$ on the closed square with vertices $(1, 1)$, $(-1, 1)$, $(1, -1)$, $(-1, -1)$.

(4) Find all the critical points of $f(x, y) = 2x^3 + 2y^3 - 3x^2y^2$, and classify them as local maxima, local minima, or saddle points. Does f attain a global maximum or a global minimum? Why or why not?

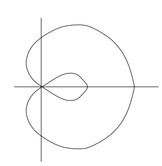
(5) A thin plate is shaped like a quarter circle of radius A and at any point p has density directly proportional to the distance from p to the corner O. (See the picture). Compute the mass of the plate and the center of mass.

(6) Find the point(s) on the graph of $x^2 + 4y^2 + 9z^2 = 1$ that are closest to the origin.

(7) The picture depicts the graph of $r = 1 + 2\cos\theta$. Find the area enclosed by the inner loop.



(5)



(7)

FIGURE 1. Pictures for problems 5 and 7

Date: Wednesday, 20 November 2002.