CALCULUS 233H EXAM I

This exam is worth 100 points, with each problem worth 20 points. Please complete Problem 1 and then *any four* of the remaining six problems. Unless indicated, you must show your work to receive credit for a solution.

When submitting your exam, please indicate which problems you want graded by writing them in the upper right corner on the cover of your exam booklet. You must select exactly four problems; any unselected problems will not be graded, and if you select more than four only the first four (in numerical order) will be graded.

(1) Please classify the following statements as *True* or *False*. Write out the word completely; do not simply write *T* or *F*. There is no partial credit for this problem, and it is not necessary to show your work for this problem.
  (a) The curvature can be computed using $\kappa(t)\mathbf{N} = d\mathbf{T}/ds$.
  (b) If $\mathbf{f}(t)$ is a vector-valued function, then $\mathbf{f}'(t)$ is always perpendicular to $\mathbf{f}(t)$.
  (c) The arc length of the space curve that is the graph of $\mathbf{f}(t)$ from $t = \alpha$ to $t = \beta$ is $\int_\alpha^\beta |\mathbf{f}(t)|\, dt$.
  (d) If $\mathbf{f}' \neq \mathbf{0}$, then the unit tangent is given by $\mathbf{f}'/|\mathbf{f}'|$.
  (e) The graph of $x^2 - y^2 - z^2 = 1$ is an ellipsoid.
  (f) The helix is the only graph of a vector-valued function with zero torsion.

(2) Let $\mathbf{f}(t) = (t\sqrt{2}, \sin t + \cos t, \sin t - \cos t)$. Compute the arc length of the graph of $\mathbf{f}$ from $t = 0$ to $t = 2\pi$.

(3) Let $\mathbf{f}(t) = (e^t, e^t \sin t, e^t \cos t)$, and let $\mathbf{f}''(t) = a_T\mathbf{T} + a_N\mathbf{N}$ be the representation of the acceleration in the tangential and normal components. Compute a_T and a_N at $t = 0$.

(4) Let $\mathbf{f}(t) = (t^3/3, t^2/2, t)$. Compute the curvature and the torsion at $t = 1$.

(5) Two lines are given by the parametric equations

$$L_1(t) = (2t, 3t + 1, 4t + 2) \quad \text{and} \quad L_2(t) = (-t + 1, t, t - 1),$$

where t is a parameter that varies over all real numbers. Determine whether or not L_1 and L_2 intersect. If they do, find the point of intersection. If they don't, determine the distance between them.

(6) Consider the three points $(1, 0, -2), (3, -1, -3), (-2, 0, 1)$.
  (a) Find the area of the triangle with these points as vertices.
  (b) Find the equation of the plane through these three points. Does this plane pass through the origin?

(7) Let $f(x, y) = e^{xy}/(x + y)$. Compute $\partial f/\partial x$ and $\partial f/\partial y$. Show that $\partial^2 f/\partial y\partial x$ and $\partial^2 f/\partial x\partial y$ are equal by computing them.

Date: Wednesday, 9 October 2002.